A STOCHASTIC MODEL FOR THE PATENT EXAMINING PROCESS*

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A. Introduction

The United States Patent Office is a complex operating system in which patent applications are received and processed according to strict rules of practice. The heart of the examining process is the patent examiner. While his primary function is that of ruling on admissibility of patent claims, the importance of his communication with the applicant in narrowing and molding the scope of claims cannot be ignored. The two-way communication between examiner and applicant (usually through the applicant's attorney) is accomplished in an environment of laws, rules, operating procedures, boards, other examiners, clerical operations and files, and these all taken together constitute the patent examining system.

Rulings of the examiner are based upon his knowledge of the art within which he is working, augmented by references he finds in the files, so that his ability to extract pertinent information from the files is one of the keys to successful operation of the system, and constitutes the search which he is required by law to make. The complexity of the search has led to interest in information storage and retrieval, the establishment of the Research and Development Office, and the initiation of research now in progress.

It is worth noting, however, that the storage and retrieval of information is only a part (admittedly an important one) of the entire system. Improvements in this activity will be reflected in the operation of other components of the system. Thus it is important to understand the operation of the entire system in order to evaluate and possibly to make conditional predictions as to the effect of changes in any of the components. One approach to describing the entire system is to develop a mathematical model of it. Such a model must be "stochastic", in the sense that one cannot predict with certainty the length of time required to perform any of the examining functions, nor whether the claims of the application will be denied, approved, or modified. However, all of the operations of the system might be well described by probability distributions so that, even though one cannot describe the course of a particular application with precision, he can, nevertheless, treat with considerable accuracy the behavior of the system under a large input of applications.

If successfully constructed, a stochastic model of the Patent Office should: 1. Serve as a basis for understanding the structure of the system. 2. Reveal where there is potential for major gains in efficiency. 3. Show the effect of proposed changes in one component on other components of the system due to interdependence of the process elements. 4. Indicate the direction of needed experimentation. 5. Tie together the pieces of the research program.

B. Construction of the Transition Model

It will be convenient to think of the system as composed of stations and activities which are performed at the stations. The stations represent locations at which functions related to a patent application take place. The application does not always pass from station to station, but the responsibility for the next action does. For example, the application is not physically returned to the applicant for amendment, but the examiner's action on the application is sent and this shifts the responsibility for the next action to the applicant.

A simplified flow diagram for actions related to patent examinations is given in Figure 1. Note that the "examiner" station is actually a collection of stations (examiners, divisions, groups or operations) and the "applicant" station can be similarly subdivided. The degree to which these stations will be subdivided will depend on the use to be made of the model. Once an application has been assigned and examined it is presumed that all future examiner actions are accomplished within the same examining organization (division or group). There may be exceptions, of course, but their importance will be determined in the process of

*The research leading to this paper was done under contract to the U. S. Patent Office with the cooperation and assistance of the staff of the Office of Research and Development. Data presented here were collected by Patent Office chemical analysts under the direction of Mr. Leibowitz of the Office of Research and Development.

collecting data for the parameters of the model.

When the application is received it is given a filing date (an important step with regard to patent rights) and is microfilmed. Certain other preliminary things are done to it and it is assigned to an examining division. Any controversy over what division should handle the application is settled by the Classification Division. Upon reaching an examining division it must await its turn among all original applications. That is, it must go to the end of the "queue" unless priority is given to it by the Commissioner of Patents. It should be noted that there are separate queues for initial applications and amended applications and no fixed rules for determining which of the two shall be given preference.

After examining the application the examiner may "allow" all or part of the claims or "reject" them. If the claims are allowed, a notice of allowance is sent to the applicant and, upon receipt of a fee by the Patent Office and completion of other mechanical details, the patent is issued. It is printed and copies are placed in the files as well as made available for sale.

If the claims are rejected the applicant is so notified and the basis for the rejection is given in the examiner's action. The applicant has a fixed time period within which to amend the application, otherwise it goes to the abandoned file. Assuming that he amends his claims, his amended application is placed at the end of the examiner's queue of amendments and is acted on after everything ahead of it (unless priority is given).

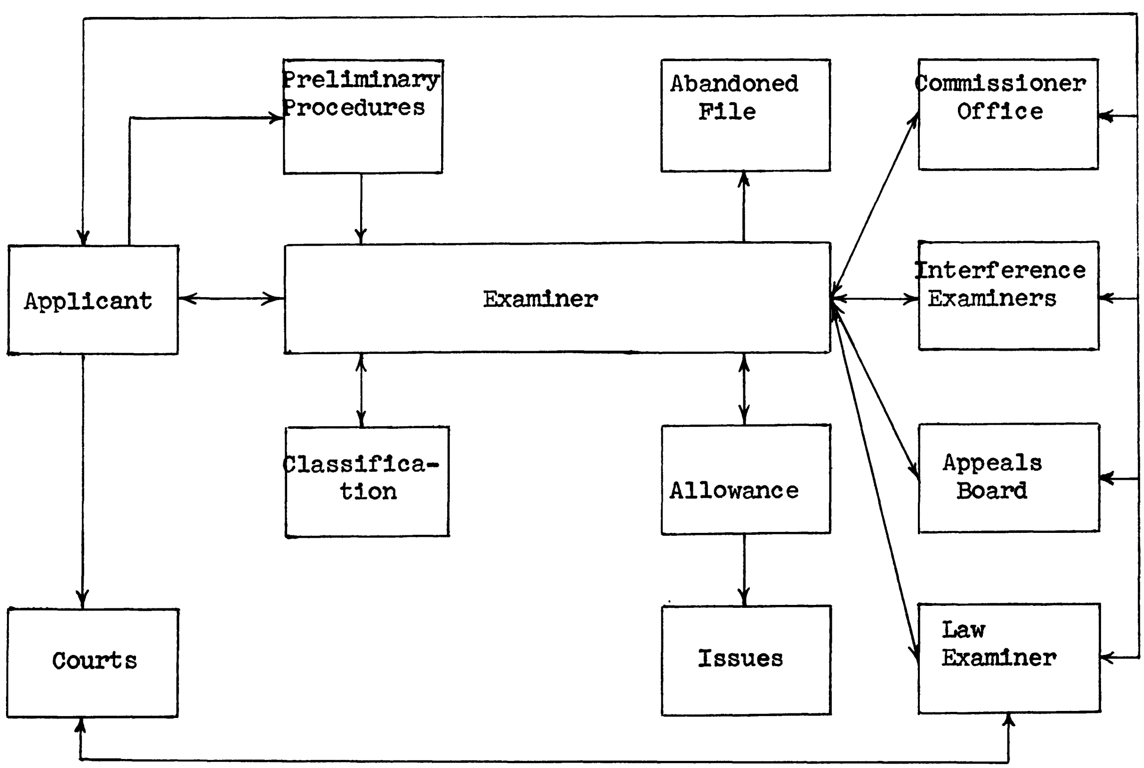

Figure 1. Flow diagram for actions in patent examinations.

After a second rejection the applicant may appeal to the Board of Appeals, he may abandon the application, or he may amend it again. Ultimately, every application is either abandoned or issued (in case any part of it is allowed), but actions on some applications follow a circuitous route before final disposition.

When an application is judged allowable by the examiner it is his responsibility to effect an "interference search" to determine whether there are other applications in process or recently issued patents whose claims are so similar as to raise questions concerning priority of invention. If, in the examiner's judgement, there are such possible interferences he will take the necessary action to set up "interference proceedings" in an effort to determine priority of invention. He may notify the law examiner who asks all parties except the one with earliest filing date for affidavits relating to evidence of date of invention. If, in the law examiner's opinion, there are still grounds for controversy the patent examiner holds an interference hearing to try to decide the matter. The examiner may also take action directly with the parties, suggesting the wording of claims so that there is no doubt that the same invention is being claimed. He may hold hearings with the parties, but if the issue cannot be resolved by him, an interference is formally declared, and the case goes to the interference examiners who make the final decision for the Patent Office. Appeal from this action can be taken only through the courts.

The function of the Appeals Board is to determine patentability controversies which arise between applicant and examiner. Again, its decision is final with respect to the Patent Office, and recourse is only through the courts.

Another action which may be taken is for the applicant to petition the Commissioner of Patents for reconsideration of an action by an examiner.

This brief outline of the examining process is vastly oversimplified. It is not intended to be comprehensive, but simply to indicate that there is a flow of actions through a network of stations, at each of which some time is required to service the application and at most of which there is a queue awaiting service. Since the flow of actions can be described in terms of probabilities, the system is called "stochastic" and it should be possible to construct a mathematical model of it.

One has a great deal of flexibility in choosing the amount of detail to be specified by the model. For example, one might wish to consider a model consisting only of applicant, examiner, issue and abandonment stations. Or he might restrict his attention to the examining process for chemical patents. Here it should be noted that the examining system is, in fact, a collection of subsystems, one for each grouping of the arts. They are tied together at certain common stations through which all may pass, such as issue, abandonment, appeal, classification, and so forth. Thus it is possible to aggregate arts in any manner which is meaningful with regard to the problem under investigation. The model which is contemplated here is general enough that it can encompass any of the specialized models which may be needed.

We first consider a transition model which attaches probabilities to the passage of actions from one station to another. It is independent of time and hence has serious limitations. A time-related model will be presented later.

For any particular form of the model we will assume a set of stations S_i, $i = 1, 2, \ldots, n$, with probabilities governing the passage of an action from S_i to S_j. These probabilities are called "transition probabilities" and a matrix of such probabilities is called a "transition matrix". An illustration is given in Figure 2. Since the entries are probabilities we have $0 \leq p_{ij} \leq 1$ for all i and j. Also, it is clear that $\sum_j p_{ij} = 1$ for all i. A matrix exhibiting these properties is called a "stochastic matrix". In case $p_{ii} = 1$ we say that state S_i is an "absorbing state". That is, once the application has reached this state it will remain there.

We will consider here only the class of stochastic matrices having a finite number of entries, although for certain applications a system with infinitely many states may prove helpful.

In the Patent Office model the probability that action on an application will pass from S_i to S_j is dependent on the history of the document prior to its arrival at S_i. Thus the model does not exhibit the independence property of a Markov chain. However, it is possible to construct a matrix of transition probabilities with the Markov property by adding stations. For example, we can let

S_1 = source of the original application

Present station	Next Station S_1	S_2	...	S_j	...	S_n
S_1	p_{11}	p_{12}	...	p_{1j}	...	p_{1n}
S_2	p_{21}	p_{22}	...	p_{2j}	...	p_{2n}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
S_i	p_{i1}	p_{i2}	...	p_{ij}	...	p_{in}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
S_n	p_{n1}	p_{n2}	...	p_{nj}	...	p_{nn}

Figure 2. Transition probabilities governing passage of actions through the system.

S_2 = original examination by the examiner
S_3 = first amendment by applicant
S_4 = review of first amendment by examiner
S_5 = second amendment by applicant

and so forth.

To avoid an infinite set of states we can choose some arbitrarily large value representing the maximum number of times a document will be amended and require that the application either be abandoned or go to appeal at that point. A reasonable value can be determined by survey, and little damage will be done to the model by imposing this kind of restriction.

C. Mathematical Notes on the Transition Model.

Consider a system in which there are absorbing states A_1, A_2, ..., A_r, and non-absorbing states T_1, T_2, ..., T_s. The total number of states is then r + s = n. The stochastic matrix of transition probabilities $P = \|p_{ij}\|$ may be partitioned as follows:

$$P = \left[\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right]$$

where I is the r x r identity matrix representing the absorbing states, R is an s x r matrix of probabilities of transition from nonabsorbing to absorbing states, Q is an s x s matrix of probabilities of transition from nonabsorbing to nonabsorbing states, and 0 is an r x s matrix of zeros. We assume that states can be chosen in such a way that the transition probabilities are dependent only on the state i, so that the stochastic matrix is a representation of a Markov chain with absorbing states. Ways of selecting states so as to achieve this result are discussed in the following section.

Let $p_{ij}^{(m)}$ be an entry of the matrix found by multiplying the matrix P by itself m times. That is $P^m = \|p_{ij}^{(m)}\|$. The entires $p_{ij}^{(m)}$ represent the probabilities of transition from state i to state j in exactly m steps. Consider the powers of the partitioned matrix, P.

$$P^2 = \left[\begin{array}{c|c} I_{rxr} & 0 \\ \hline (I_{sxs} + Q)R & Q^2 \end{array}\right]$$

In general,

$$P^m = \left[\begin{array}{c|c} I_{rxr} & 0 \\ \hline (I_{sxs}+Q+Q^2+\ldots+Q^{m-1})R & Q^m \end{array}\right]$$

If for each transition state i there exists an integer k such that $p_{ij}^{(k)} > 0$, where j is an absorbing state, Q^m will converge to the null matrix and $(I + Q + Q^2 + \ldots + Q^{m-1})$ will converge to the inverse* $(I - Q)^{-1}$. The matrix $(I-Q)^{-1}$ is called the "fundamental matrix" of the Markov chain.

Consider a nonabsorbing state i and an absorbing state j. Let q_{ik} denote an entry of the matrix Q and r_{ij} denote an entry of the matrix R. Then the probability of transition from i to j in one step is r_{ij}, in two steps is $\sum_k q_{ik}r_{kj}$, in three steps is $\sum_m\sum_k q_{ik}q_{km}r_{mj}$, and so forth. These are seen to be the entries of the matrices R, QR and Q^2R. Thus, the partitioned form of P^m, above, shows that the probability of ultimate transition from i to j can be found by the entries of the matrix B, where

$$B = (I - Q)^{-1}R.$$

Hence, one can readily determine probability of absorption in any absorbing state from any nonabsorbing state.**

Also, the expected number of times of being in state k (a nonabsorbing state), starting from state i (another nonabsorbing state), is the ikth entry of $(I-Q)^{-1}$. We are primarily interested in the first row of the inverse which shows the expected number of times a patent application will be in each state, starting from its receipt by the Patent Office. By adding together the entries for all examiner actions we can determine the expected number of examiner actions required.

*David Rosenblatt, "On linear models and the graphs of Minkowski-Leontief matrices", Econometrica, Vol. 25, No. 2, April, 1957, pp. 325-28, shows that the inverse will exist for substochastic matrices Q if such Q contain no stochastic closed cyclic nets.

**These results may all be found in Kemeny and Snell, Finite Markov Chains, D. Van Nostrand Co., Inc., Princeton, 1960, Chapter III.

D. Illustration of the Transition Model with Actual Data from the Patent Office.

The data. A random sample of 500 cases relating to chemical patents was drawn from those on which action had been completed in 1962. The sample was allocated proportionately between abandoned and allowed cases so that results could be combined without weighting. The study was restricted to chemical patents because the Chemical Operation* was the only one of the four Patent Office examining operations which had been reorganized at the time the sample was drawn. Also, other research was in progress in the Chemical Operation which made it possible to use the sample for multiple purposes.

Each case is represented by a file containing the original application, the examiners' actions, amendments to the application, notices of appeals, briefs, arguments, and all ancillary papers associated with the process of obtaining (or being denied) a patent. All papers are dated so that elapsed time from one step to another can be computed. Examination of the sequence of actions identifies the "states" which must be considered in the stochastic matrix of transition probabilities.

The data collection forms, as well as a discussion of the various contemplated models, have been presented in an informal report** to the Patent Office and will not be reproduced here. It is sufficient to say that the required data were extracted from the files by trained chemical analysts in the Office of Research and Development, and were then punched on IBM cards for flexibility in analysis.

The states and transition probabilities.***

*The word "Operation" refers to a major grouping of the arts for administration purposes.

**Bryant, E. C., "On stochastic models of the Patent Office examining system," WRA PO 8, April, 1963, Office of Research and Development, United States Patent Office.

***All of the numerical data in this paper must be considered to be preliminary, since the limited time available for the preparation of this paper has made it necessary to use data which have not been checked.

The preliminary model contains the following states:

000 - original application

001 - first amendment prior to final rejection

002 - second amendment prior to final rejection, etc.

010 - first amendment after final rejection

011 - second amendment after final rejection, etc.

021 - first amendment after notice of appeal

022 - second amendment after notice of appeal, etc.

100 - examiner action on original application

101 - examiner action in first amendment, etc.

111 - examiner action on first amendment after final rejection

112 - examiner action on second amendment after final rejection, etc.

312 - amendment after allowance

313 - request for amendment after allowance denied

320 - certificate of correction (after issue)

401 - first examiner amendment

402 - second examiner amendment

411 - first examiner amendment after final rejection

412 - second examiner amendment after final rejection

500 - final rejection after original application (can only occur when application is a continuation in part of a previous action)

501 - final rejection after first amendment, etc.

700 - notice of appeal

701 - appealed case settled by examiner

702 - appealed case decided by Board of Appeals

703 - appealed case decided by Board, with subsequent action by examiner (on amended claims)

751 - interference proceedings

752 - interference case settled by examiner

753 - interference case settled by Board of Interference Examiners

754 - interference case settled by Board, with subsequent action by examiner (on amended claims, or claims not in interference)

760 - both interference and appeal

800 - allowance (subject to amendment)

860 - final issue

900 - abandonment

The matrix of transition probabilities is a 48 x 48 matrix and hence is too extensive to reproduce here except in abbreviated form. Since most of the entries are zeros it is possible to express the matrix as shown in Table 1. The decision to use the particular states shown is an arbitrary one. It is possible, for example, to construct a simplified model in which one does not distinguish between the various kinds of amendments or examiner actions. The states chosen are believed to be adequate to develop the kind of analysis needed for initial management decisions.

It should be pointed out that a single state of this model may, in fact, represent a sequence of operations which can be expressed as another stochastic model. As an example, consider the actual process of acting on an application. One might consider the states of (1) preliminary reading and study, (2) literature search, (3) detailed study, (4) writing the action, (5) typing, (6) reading and signature, and (7) mailing. Undoubtedly, other states would reveal themselves during the analysis of the data. Some research is planned in the development of such detailed model if the larger model, presented here, appears to yield useful results.

The states in the matrix of Table 1 have been arranged in such a manner that the matrix is triangular, that is, no entries appear below the main diagonal. This was accomplished by careful choice of the transition states. A model in which one does not distinguish between a first amendment and a second amendment or between a first and second action would not have this property.

The matrix of Table 1 was partitioned and the fundamental matrix $(I-Q)^{-1}$ computed, as well as the product of this matrix and the matrix R (transition from nonabsorbing to absorbing states). Some of the pertinent results are shown in Table 2. The first column shows probability of issue of a patent, given that the application is in the state indicated in the margin. Note that these are probabilities of issuing <u>a</u> patent, not necessarily on all of the claims currently being made. For example, when a case is appealed (state 700) the probability that a patent will be granted is .662, yet the annual report of the Commissioner of Patents for FY 1962 shows that the examiners' decisions were reversed in whole or in part in only 21 out of 81 cases. This apparent discrepancy is due to two principal factors: (1) most cases for

Table 1. Matrix of transition probabilities, Chemical Operation examining model.

Origin	No. of obsns.	Destination states and transition probabilities (given in parenthesis)			
000	491	100 (.917)	001 (.065)	500 (.002)	401 (.004)
		800 (.008)	900 (.004)		
100	452	001 (.910)	101 (.002)	751 (.004)	900 (.084)
001	443	101 (.679)	002 (.065)	501 (.171)	751 (.006)
		401 (.025)	800 (.052)	900 (.004)	
101	304	002 (.921)	501 (.003)	751 (.008)	800 (.003)
		900 (.065)			
002	313	102 (.403)	003 (.102)	502 (.358)	401 (.029)
		800 (.102)	900 (.006)		
102	126	003 (.921)	700 (.016)	401 (.008)	900 (.055)
003	152	103 (.230)	004 (.145)	503 (.474)	751 (.007)
		401 (.039)	800 (.105)		
103	35	004 (.829)	900 (.171)		
004	50	104 (.200)	005 (.080)	504 (.500)	760 (.040)
		401 (.060)	800 (.120)		
104	10	005 (.800)	800 (.100)	900 (.100)	
005	13	105 (.077)	006 (.154)	505 (.538)	800 (.231)
105	1	006 (1.000)			
006	4	106 (.250)	506 (.750)		
106	1	900 (1.000)			
500	1	700 (1.000)			
501	77	011 (.623)	760 (.013)	700 (.182)	900 (.182)
502	112	011 (.652)	760 (.018)	700 (.125)	900 (.205)
503	72	011 (.528)	700 (.208)	900 (.264)	
504	25	011 (.760)	700 (.160)	900 (.080)	
505	7	011 (.857)	900 (.145)		
506	3	011 (1.000)			
011	182	111 (.330)	012 (.115)	751 (.027)	700 (.176)
		401 (.010)	411 (.045)	800 (.280)	900 (.017)
111	61	012 (.459)	700 (.377)	411 (.016)	900 (.148)
012	46	112 (.173)	013 (.174)	760 (.022)	700 (.196)
		401 (.174)	800 (.261)		
112	4	013 (.500)	700 (.250)	900 (.250)	
013	13	113 (.077)	014 (.077)	401 (.383)	800 (.461)
113	1	800 (1.000)			
014	1	800 (1.000)			
751	9	752 (.111)	753 (.333)	754 (.556)	
752	1	800 (1.000)			
753	3	800 (.667)	900 (.333)		
754	5	800 (.200)	900 (.800)		
760	6	800 (.667)	900 (.333)		
700	101	701 (.634)	702 (.267)	703 (.099)	
701	64	800 (.953)	900 (.047)		
702	27	800 (.037)	900 (.963)		
703	10	800 (.600)	900 (.400)		
401	34	402 (.029)	800 (.971)		
402	1	800 (1.000)			
411	25	412 (.040)	800 (.920)	900 (.040)	
412	1	800 (1.000)			
800	294	312 (.118)	313 (.007)	850 (.861)	900 (.014)
312	33	850 (.970)	900 (.030)		
313	2	850 (1.000)			
850	293	320 (.014)	860 (.986)		
900		900 (1.000)			
860		860 (1.000)			

which a notice of appeal is filed are actually settled by the examiner, and (2) cases which are decided by the Board against the appellant may contain claims which can be allowed.

The second column of Table 2 permits computation of the expected number of times an action is taken by each participant in the examination process. Adding together the entries for 101-606, 500-06, 111-114, 401-2, 411-12, 312-313, and 800 yields 3.40, an estimate of the number of times an examiner is required to act on the claims in an application, exclusive of interference and appeals cases. A study of appeals cases (resulting in a separate model) shows that an examiner is required, on the average, to act on each appealed case about 1.8 times. Since about one-fifth of the cases go to appeal, the average number of examiner actions is about 3.75. It is important to note that there is a high degree of arbitrariness in the definition of an examiner action.

Table 2. Selected results from analysis of the chemical operation transition model.

Identification		Prob. of issue	Prob. of occupancy
000	original application	.588	1.000
100	examiner action	.584	.917
001	first amendment	.640	.899
101	examiner action	.612	.613
002	2nd amendment	.655	.621
102	examiner action	.617	.250
003	3rd amendment	.651	.231
103	examiner action	.612	.075
004	4th amendment	.740	.098
104	examiner action	.671	.020
005	5th amendment	.716	.024
105	examiner action	.578	.002
006	6th amendment	.578	.005
106	examiner action	0*	.002
500	final rejection 000	.662	.002
501	final rejection 001	.607	.155
502	final rejection 002	.597	.222
503	final rejection 003	.544	.139
504	final rejection 004	.692	.049
505	final rejection 005	.661	.013
506	final rejection 006	.770	.005
011	1st amend. after final	.770	.368
111	examiner action	.658	.122
012	2nd amend. after final	.855	.098
112	examiner action	.655	.017
013	3rd amend. after final	.981	.026
113	examiner action	.981	.002
014	4th amend. after final	.981	.002
751	interference	.436	.027
752	decided by examiner	.981	.003
753	decided by board	.656	.008
754	decided by board and examiner	.196	.015
760	interference and appeal	.655	.013
700	appeal	.662	.232
701	decided by examiner	.936	.155
702	decided by board	.036	.063
703	decided by examiner and board	.589	.024
401	examiner amendment	.981	.094
402	examiner amendment	.981	.003
411	examiner amendment	.942	.018
412	examiner amendment	.981	.001

*Based on a single case

(continued)

800	allowance	.981	.601
312	amendment	.970	.071
313	amendment not entered	1.000	.005
850	issue	1.000	.591
320	correction	1.000	.006

E. Time Dependent Models

The model presented in previous sections permits one to trace the flow of actions through the network, but at no place does time enter as a parameter. Thus, no conclusions can be drawn concerning length of time required to process an application to final disposition, number of examiners required to staff a collection of arts, or size of the backlogs at a given time. These are all matters of the utmost importance to the Patent Office, so that a model which includes time is essential.

A model with fixed time increments. A single formulation of the model is obtained by tracing the location of the application (or action) during each successive time increment (day, week, half day, etc.). The basic idea is presented in Figure 3.

The probability that an action will be in state S_k at time t_p is the sum of the probabilities of achieving that state at time t_p by all possible routes. For example, the probability that an action, starting at S_1 at time t_1, will be at S_3 at time t_4 is equal to $(.9)(.2)(.2) + (.1)(.7)(.2) + (.1)(.3)(.8) = .074$.

It is clear that Figure 3 can be expressed in matrix form by assigning a symbol to each node in the graph and assigning to each entry of the matrix the probability of transition from one node to another. This representation appears in Table 3. To prevent confusion, the word "state" will be used to denote the S_k and "node" will be used to denote combinations of S_k and t_p.

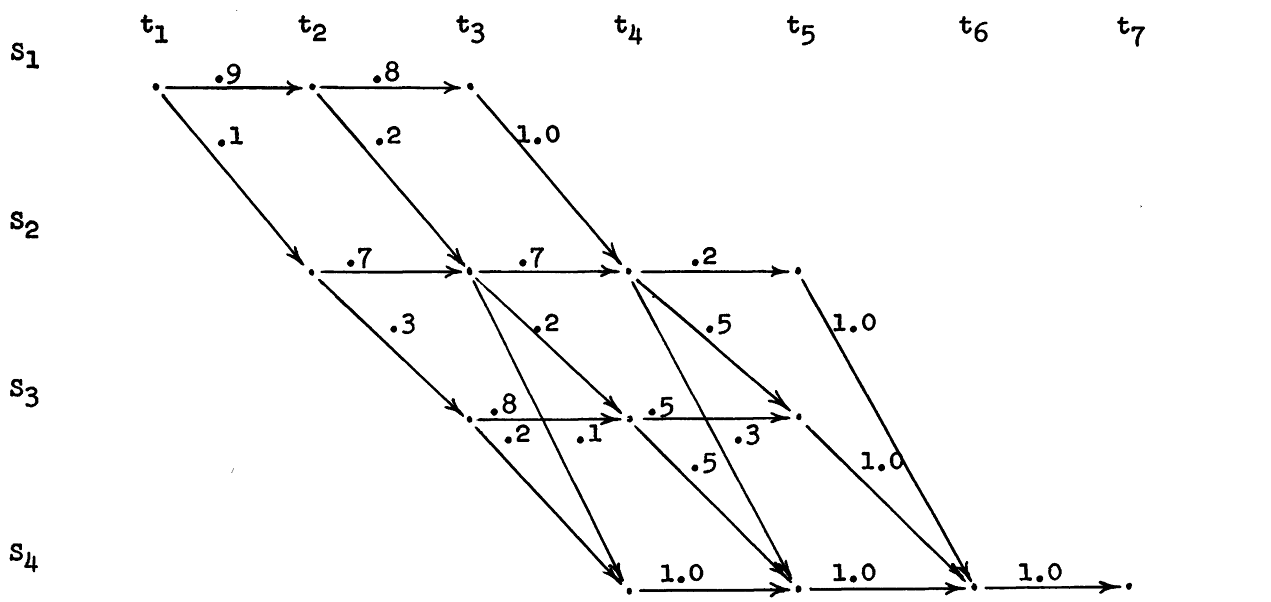

Figure 3. Flow of actions during fixed time increments.

Table 3. Matrix of the transition probabilities of Figure 3.

	S_1t_1	S_1t_2	S_1t_3	S_2t_2	S_2t_3	S_2t_4	S_2t_5	S_3t_3	S_3t_4	S_3t_5	S_4t_4	S_4t_5	S_4t_6	S_4t_7
S_1t_1	0	.9	0	.1	0	0	0	0	0	0	0	0	0	0
S_1t_2	0	0	.8	0	.2	0	0	0	0	0	0	0	0	0
S_1t_3	0	0	0	0	0	1	0	0	0	0	0	0	0	0
S_2t_2	0	0	0	0	.7	0	0	.3	0	0	0	0	0	0
S_2t_3	0	0	0	0	0	.7	0	0	.2	0	0	0	0	0
S_2t_4	0	0	0	0	0	0	.2	0	0	.5	0	.3	0	0
S_2t_5	0	0	0	0	0	0	0	0	0	0	0	0	1	0
S_3t_3	0	0	0	0	0	0	0	0	.8	0	.2	0	0	0
S_3t_4	0	0	0	0	0	0	0	0	0	.5	0	.5	0	0
S_3t_5	0	0	0	0	0	0	0	0	0	0	0	0	1	0
S_4t_4	0	0	0	0	0	0	0	0	0	0	0	1	0	0
S_4t_5	0	0	0	0	0	0	0	0	0	0	0	0	1	0
S_4t_6	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S_4t_7	0	0	0	0	0	0	0	0	0	0	0	0	0	1

An absorbing node, S_4t_7, has been introduced to close the system. It might as well have been introduced as node S_4t_6. The first row of elements u_{kp} of the inverse $(I-Q)^{-1}$ follows, where the first subscript denotes the state and second the time increment:

u_{11} =	1.0000	u_{33} =	.0300
u_{12} =	.9000	u_{34} =	.0740
u_{13} =	.7200	u_{34} =	.4845
u_{22} =	.1000	u_{44} =	.0310
u_{23} =	.2500	u_{45} =	.3365
u_{24} =	.8950	u_{46} =	1.0000
u_{25} =	.1790		

The average number of time increments (and hence the average time) in state S_k is obtained by summing on the second subscript. Thus the average time in state S_2 is obtained by

$$\sum_p u_{2p} = 1.424 \text{ time units.}$$

All actions will have reached state S_4 after five time periods. The average time required to reach state S_4 (the absorbing state) can be determined from average times in each node associated with state S_4, i.e., S_4t_4, S_4t_5, and S_4t_6. The average times in node S_4t_4 is, in fact the probability of reaching S_4 in three steps. The probability of reaching S_4 in 4 steps is the average times in S_4t_5 minus the average times in S_4t_4, and so forth. Thus the average time required to reach the absorbing state S_4 is

$$3u_{44} + 4(u_{45} - u_{44}) + 4(u_{46} - u_{45}) =$$
$$3(.031) + 4(.3255) + 5(.6635) =$$
$$4.6325 \text{ time units.}$$

The extension to more complex cases is obvious.

One of the real difficulties with this approach is that the matrix dimensions become quite large. For example, if one uses an average of 15 time units for each state and there are 50 states in the model, a 750 x 750 transition matrix will result. However, with some minor adjustments it can be made triangular. Furthermore, most of the entries are zeros. A method has been developed for manipulating transition matrices of finite Markov chains which depends on the fact that they can be decomposed into linear functions of deterministic matrices (all entries zeros or ones).* We feel that this approach is worth investigating, although there has been insufficient time to try out the method in the preparation of this report.

Time distributions have been developed for the length of elapsed time between all of the pairs of states in the model previously presented. A few are shown in Tables 4 and 5. These tables indicate that one must distinguish between first, second, third, etc., actions and between destinations (i.e., next state). They also reflect the six months statutory limitation on time allowed for amendment. The Patent Office time distributions clearly reflect priorities given to final actions, usually for purposes of appeal. Time distributions between final rejection and appeal are now shown, but they very closely approach the six months statutory period.

A partial list of transition states and probabilities is shown in Table 6.

Table 4. Frequency distributions of elapsed time between receipt of application (or amendment) and examiner action (time in Patent Office).

Days	Original Application	First Amendment Not Final	First Amendment Final	Second Amendment Not final	Second Amendment Final	Third Amendment Not final	Third Amendment Final
0- 29	0	6	11	14	21	7	15
30- 59	11	3	4	15	6	1	11
60- 89	20	19	4	8	12	2	4
90-119	33	12	6	2	12	5	6
120-149	39	26	7	14	9	4	7
150-179	60	41	5	16	7	2	6
180-209	77	44	9	16	10	0	8
210-239	74	55	5	18	9	3	6
240-269	55	41	7	9	8	2	6
270-299	43	24	7	4	6	4	2
300-329	24	13	3	6	6	4	1
330-359	6	7	3	3	6	1	1
360-389	4	1	3	0	1	1	0
390-419	4	3	1	1	1	0	0
420-449	0	3	0	0	0	0	0
450 and over	0	4	0	1	0	0	0

Table 5. Frequency distributions of elapsed time between mailing of rejection and receipt of further amendment (time in hands of applicant).

Days	First Action	Second Action Not final	Second Action Final	Third Action Not final	Third Action Final	Fourth Action Not final	Fourth Action Final
0- 29	22	11	8	23	12	0	27
30- 59	12	8	5	3	4	12	7
60- 89	13	10	4	13	12	7	2
90-119	10	13	7	7	8	0	5
120-149	24	20	7	12	7	4	8
150-179	192	122	8	38	21	6	7
180-209	138	95	5	32	5	6	2
210-239	0	0	0	1	1	0	0
240-269	1	0	0	2	0	0	0
270 and over	1	0	0	2	0	0	0

Machine simulation of the system. The estimated probabilities of Table 1 and the time distributions, of which Tables 4 and 5 are samples, make it possible to simulate the system by electronic computer. For each input to the system (new application) one selects, by drawing random numbers proportional to transition probabilities, a path through the system. The delay at each station is determined by drawing random numbers proportional to

*See S. C. Gupta, "Manipulation of state transition matrices of finite Markov chains," IEEE Transactions on Electronic Computers, February 1963, pp. 19-20.

the time-frequency distributions for some suitable time interval, perhaps 20 or 30 days. The remainder of the program is an accounting scheme to keep track of applications during each increment of time. One must also enter data reflecting the current status of the operation, including applications in the backlog, in the hands of applicants awaiting amendment, in the Board of Appeals, etc. These data are available from administrative data of the Patent Office and from the time distributions, above.

After entry of the required estimated parameters, one simulates the operation of the system for a sufficient time to insure that his estimates do, in fact, reflect the behavior of the system. He then may vary some of the parameters, such as number of examiners, changes in statutory time limits, and so forth, to observe the effect on output and backlogs of these changes. The data which have been gathered and the preliminary analysis of the data indicate that the system can be simulated. If so, the potential of the simulation model for decision-making purposes is obvious.

Table 6. Partial list of transition states and probabilities (in parenthesis) for the transition model with fixed time increments of 30 days.

Present station	Next station (and probability)			
000.1	000.2 (.966)	001.1 (.034)		
000.2	800.1 (.957)	001.1 (.015)	100.1 (.022)	500.1 (.002)
	800.1 (.004)			
000.3	000.4 (.956)	001.1 (.004)	100.1 (.040)	
000.4	000.5 (.927)	001.1 (.006)	100.1 (.067)	
000.5	000.6 (.915)	001.1 (.002)	100.1 (.080)	900 (.003)
000.6	000.7 (.876)	100.1 (.122)	401.1 (.001)	800.1 (.001)
000.7	000.8 (.843)		100.1 (.157)	
000.8	000.9 (.846)	001.1 (.002)	100.1 (.150)	401.1 (.001)
	900.1 (.001)			
000.9	000.10(.887)	100.1 (.112)	800.1 (.001)	
000.10	000.11(.910)	001.1 (.002)	100.1 (.088)	
000.11	000.12(.951)	100.1 (.049)		
000.12	000.13(.988)	100.1 (.012)		
000.13	100.1 (.999)	800.1 (.001)		
001.1	001.2 (.935)	002.1 (.021)	101.1 (.044)	
001.2	001.3 (.965)	002.1 (.013)	101.1 (.022)	
001.3	001.4 (.857)	002.1 (.002)	101.1 (.141)	
001.4	001.5 (.912)	101.1 (.088)		
001.5	001.6 (.704)	002.1 (.100)	101.1 (.192)	751.1 (.004)